Error mitigation protocol for multi-photon noise in quantum repeater chains

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Abstract: Noise from multi-photon emissions can severely limit the use of probabilistic photon sources in quantum repeater chains. We present a Bell state measurement protocol that suppresses noise from multi-pair emissions in quantum repeaters.

1. Alternating Bell state measurement (ABSM) protocol

It was shown by Guha *et al.* that multi-pair emissions can severely limit useful application of probabilistic photon sources in a quantum repeater architecture [1]. This constrains the use of parametric down-conversion sources—as well as atomic ensembles in a DLCZ-type scheme [2]—in an extended repeater chain. Ideal photon number resolving (PNR) detectors can potentially overcome this limitation [3]; however, effective PNR requires highly efficient repeaters, since any photon losses between multi-photon production and detection limit the ability of the detector to identify multi-photon emissions. We introduce a protocol for mitigating multi-photon noise in repeater chains which does not rely on PNR and supplements the partial mitigation afforded by imperfect PNR.

The ABSM protocol exploits correlations present in the 4-photon emission in a polarization-entangled two-mode squeezed vacuum (TMSV) state in a way that prevents a double-pair emission from independently triggering two adjacent Bell state measurements (BSMs). By alternating the BSM basis in a repeater chain, a stimulated 4-photon emission from a single source cannot independently trigger two adjacent BSMs (Figure 1).

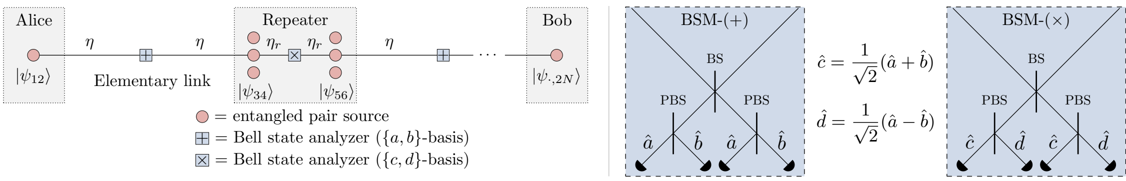

Fig. 1. Protocol for concatenated entanglement swapping links. By alternating the BSM measurement basis, a double-pair emission from a single source cannot trigger two adjacent BSMs.

To make this rigorous, we express the entangled photon state–representing a pair of dual-rail qubits in a superposition of modes with bosonic annihilation operators $\{a_1, b_1\}$ and $\{a_2, b_2\}$—in the form [1]

$$|\psi_{12}\rangle = \sqrt{p_{12}^{(0)}}|0\rangle + \sqrt{p_{12}^{(1)}/2}\left(a_1^\dagger b_2^\dagger - b_1^\dagger a_2^\dagger\right)|0\rangle + \sqrt{p_{12}^{(2)}/12}\left(a_1^{\dagger 2} b_2^{\dagger 2} + b_1^{\dagger 2} a_2^{\dagger 2} - 2a_1^\dagger b_1^\dagger a_2^\dagger b_2^\dagger\right)|0\rangle \tag{1}$$

where we have truncated higher order terms in order to focus on the leading order contribution to the multi-photon error. This form for the entangled pair state is derived from a tensor product of two copies of a TMSV state. The probability of producing n pairs is given by $p_{12}^{(n)} = (n+1)(1-|\lambda|^2)^2|\lambda|^{2n}$where the parameter $|\lambda|^2 \ll 1$ determines the single-pair emission probability $p_{12}^{(1)} \simeq 2|\lambda|^2$. Correlations in the 4-photon term arise from the stimulated emission process captured in the bosonic relation $a^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$. Expanding modes $\{a_2, b_2\}$ in the diagonal basis $c_2 = \sqrt{1/2}(a_2+b_2)$ and $d_2 = \sqrt{1/2}(a_2-b_2)$ yields the 4-photon term

$$b_1^{\dagger 2}\left(c_2^{\dagger 2} + d_2^{\dagger 2} + 2c_2^\dagger d_2^\dagger\right)|0\rangle + a_1^{\dagger 2}\left(c_2^{\dagger 2} + d_2^{\dagger 2} - 2c_2^\dagger d_2^\dagger\right)|0\rangle - 2a_1^\dagger b_1^\dagger\left(c_2^{\dagger 2} - d_2^{\dagger 2}\right)|0\rangle. \tag{2}$$

The key property is that a measurement of opposite polarizations a_1, b_1 in the first output channel projects the state in the second output channel onto a correlated 2-photon N00N state in modes c_2, d_2 (and vice-versa).

One can exploit this correlation in a concatenated entanglement swap by noting that a linear optical BSM between two adjacent sources $|\psi_{12}\rangle|\psi_{34}\rangle$ only succeeds if opposite polarizations are detected. Furthermore, each BSM can be performed in either basis, as follows from the easily verified relations

$$|\Psi^+\rangle_{ab} = |\Phi^-\rangle_{cd}, \qquad |\Phi^+\rangle_{ab} = |\Phi^+\rangle_{cd}, \qquad |\Psi^-\rangle_{ab} = -|\Psi^-\rangle_{cd}, \qquad |\Phi^-\rangle_{ab} = |\Psi^+\rangle_{cd}, \tag{3}$$

where $|\Psi^{\pm}\rangle_{ab}, |\Phi^{\pm}\rangle_{ab}$ and $|\Psi^{\pm}\rangle_{cd}, |\Phi^{\pm}\rangle_{cd}$ represent the Bell states in the $\{a,b\}$ basis and $\{c,d\}$ basis, respectively. Thus, by performing adjacent BSMs in concatenated entanglement swapping links in alternating diagonal bases—as illustrated in Figure 1—the correlations observed in (2) ensure that a double-pair emission from a single source cannot independently trigger an erroneous BSM in two adjacent Bell state analyzers.

The protocol does not prevent multi-photon errors that occur when one of the photons in the double-pair state is lost. In this case, the chain can still be corrupted if a photon from an adjacent source arrives at the BSM where the secondary emission was lost. Nevertheless, the analysis described below shows that the ABSM protocol eliminates the dominant quadratic contribution to multi-photon errors in extended repeater chains.

2. Error mitigation performance for balanced repeater chains

To demonstrate the error mitigation performance, we consider a quantum repeater chain with $\ell = N/2$ balanced elementary links with N sources. By considering only the leading order errors consisting of at most $N+1$ photon pair emissions from N sources, a closed form expression for the Bell state fidelity is obtained in the form

$$F = \frac{1 + p\left(\varepsilon_+ + \frac{11+\sigma}{5+\sigma}\varepsilon_0\right)}{1 + 4p(\varepsilon_0 + \varepsilon_0' + \varepsilon_+ + \varepsilon_+')}, \tag{4}$$

where $p = p_{ij}^{(1)}$ is the source emission probability and the multi-pair emissions yield the error terms

$$\begin{aligned} \varepsilon_0' &= (1-\eta), & \varepsilon_+' &= \frac{1}{8}(1+\sigma)[2+\sigma(N-4)](1-\eta) \\ \varepsilon_0 &= \frac{N-2}{4}(5+\sigma)(1-\eta)(1-\eta_r), & \varepsilon_+ &= \frac{N-4}{16}(1+\sigma)^2[4+\sigma(N-6)](1-\eta)(1-\eta_r) \end{aligned} \tag{5}$$

with $\sigma = 1$ for the standard BSM protocol and $\sigma = 0$ for the ABSM protocol (setting $\varepsilon_0 = \varepsilon_+ = \varepsilon_+' = 0$ for $\ell = 1$).

The fidelity is essentially independent of the channel efficiency η for $\eta \ll 1$ since we can use the approximation $(1-\eta) \simeq 1$. The fidelity produced by the balanced chain thus depends only on the repeater efficiency η_r and the number of elementary links ℓ. This dependence is shown in Figure 2 for repeaters with $\eta_r = 0.9$. The corresponding gain in the elementary link efficiency is obtained by inverting (4) for the allowed emission probability $p(F;\sigma)$ at fixed fidelity F. The resulting gain $G = p(F;\sigma=0)^2/p(F;\sigma=1)^2$ is shown in Figure 2 for $F = 0.95$.

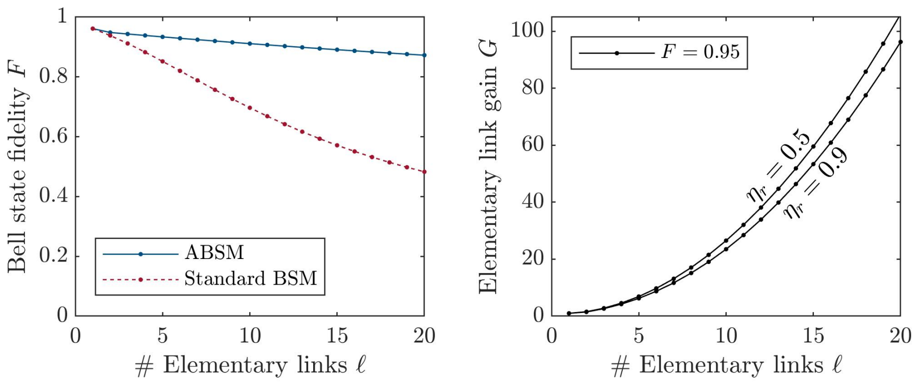

Fig. 2. The left panel shows the Bell state fidelity for repeater chains using probabilistic sources with pair probability $p = 0.01$ assuming PNR detection with repeater efficiency $\eta_r = 0.9$. The right panel shows the gain in elementary link efficiency afforded by the ABSM protocol at fixed fidelity F.

It follows from (4) that the protocol eliminates the dominant ℓ^2 growth of errors in chains with ℓ elementary links—contained in the σN^2 term in ε_+— resulting in errors which can only accumulate linearly in ℓ. Correspondingly, the gain from the ABSM protocol grows quadratically in ℓ reaching ~100-fold gain for $\ell = 20$.

References

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